Note on Block Conductor Solenoid

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Abstract

We give the expressions as power series expansion in the radial variable r, for the magnetic field of a solenoid modelled as a block conductor. This approach will be incorporated in ICOOL [1] as one of the possible options.

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I. INTRODUCTION

The reasons to use BLOCKS were nicely summarized in a recent paper [2] The exact analytical field on axis of a block conductor solenoid of inner R_1 and outer R_2 radius, total length $2 * L$ and current density J is,

$$B_s(0, s) = \frac{\mu_0 J}{2} \left\{ (s - L) Ln \frac{R_2 + \sqrt{R_2^2 + (s - L)^2}}{R_1 + \sqrt{R_1^2 + (s - L)^2}} - (s + L) Ln \frac{R_2 + \sqrt{R_2^2 + (s + L)^2}}{R_1 + \sqrt{R_1^2 + (s + L)^2}} \right\} \equiv f^{(0)}(s);$$

(1)

the meaning of $f^{(0)}(s)$ is explained below.

Knowing the field on axis it is possible to write the field at any r as a series power expansion in r. The result for the radial field $B_r(r, s)$ is

$$B_r(r, s) = -\frac{1}{2} \sum_{j=0}^{\infty} \frac{(-)^j f^{(2j+1)}(s)}{4^j j!(j + 1)!} \times r^{2j+1}$$

(2)

and for the longitudinal field $B_s(r, s)$ is,

$$B_s(r, s) = \sum_{j=0}^{\infty} \frac{(-)^j f^{(2j)}(s)}{4^j (j!)^2} \times r^{2j}$$

(3)

where $f^{(2j)}(s)$ is the 2j-derivative respect to the axial variable s of the field on axis, *i.e* Eq. 1 (see also [3]).

It is not difficult to show that these expressions satisfy $\nabla \vec{B} = 0$ ($\frac{\partial B_r}{\partial r} + \frac{B_r}{r} + \frac{\partial B_s}{s} = 0$) and $\nabla \times \vec{B} = 0$ ($\frac{\partial B_r}{\partial s} = \frac{\partial B_s}{\partial r}$) term by term.

II. DERIVATION

In cylindrical coordinates the most general scalar potential for $r < R_1$ is

$$\phi(r, s) = -\int_{-\infty}^{+\infty} d\, k e^{iks} I_0(kr) A(k)$$
$$B_r(r, s) = -\frac{\partial \phi}{\partial r} = \int_{-\infty}^{+\infty} d\, k k e^{iks} I_1(kr) A(k)$$
$$B_s(r, s) = -\frac{\partial \phi}{\partial s} = \int_{-\infty}^{+\infty} d\, k i k e^{iks} I_0(kr) A(k)$$

(4)

where $I_0(z), I_1(z)$ are the modified Bessel functions of order 0 and 1, respectively, and $A(k)$ is a coefficient to be determine by the boundary condition given by Eq. 1.

Multiplying by $\int d\, s' e^{-iks'}$ both sides of the expression for $B_s(0, s)$ we get,

$$A(k) = \frac{1}{2\pi i k} \int_{-\infty}^{+\infty} d\, s' e^{-iks'} f^{(0)}(s').$$

2

After substitution in Eqs. 4 and using the power series expansion of the Bessel functions,

$$I_0(z) = \sum_{j=0}^{+\infty} \frac{z^{2j}}{4^j (j!)^2} \qquad , \qquad I_1(z) = \frac{1}{2} z \sum_{j=0}^{+\infty} \frac{z^{2j}}{4^j j!(j+1)!} \tag{5}$$

we obtain the final results Eqs. 2 and 3.

Notice that the expressions are valid for $r < R_1$, and that the derivatives have been symbolically computed using the program *Mathematica* [4] to the 16-th order. These functions are given as external Fortran functions in ICOOL.

Next we compare the values of B_r and B_s with the values calculated using a numerical integration of sheets; both methods give identical values for the fields, except at $s = L$, $r = R_1$, at the edge of the current block distribution.

Acknowledgments

[1] R. Fernow, *ICOOL: A simulation code for Ionization Cooling of Muon Beam*, Proc. Particle Accelerator Conference, 1999.

[2] R. Fernow, *Modeling solenoids using,coil,sheet and block conductors*, MUC-NOTE-COOL-THEORY-281, Nov.3, 2003.

[3] J. Gallardo, *Note on Vector Potential and Magnetic Field for a Solenoid*, BNL- Muon Tech. Note MU-040, 2000.

[4] S. Wolfram, *Mathematica, A System for Doing Mathematics by Computer*, 2nd Edition, (Addison-Wesley, Redwood City, CA, 1991.
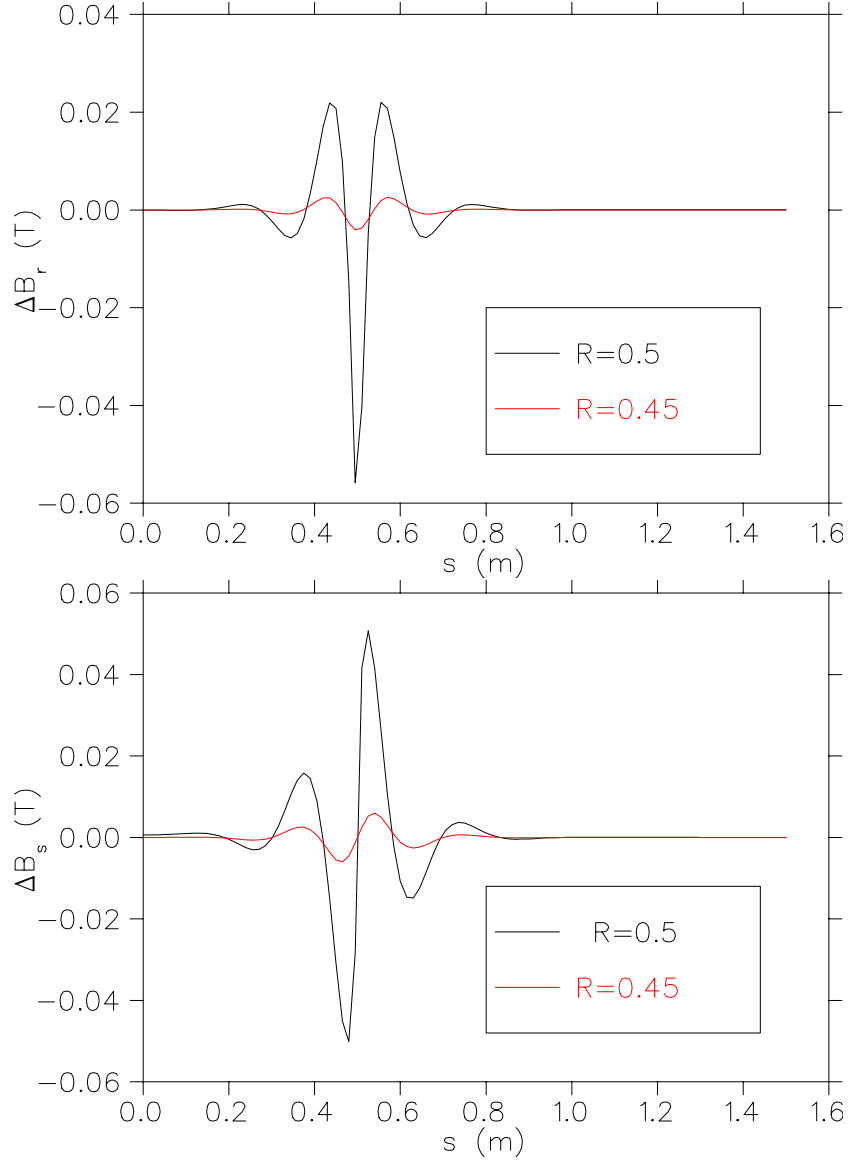
FIG. 1: Block solenoid with R1=0.5 cm, R2=0.6 cm and L=1 m. (left) B_r and (right) B_s comparison with the sheet model results.